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PROBABILITIES OF RUIN WHEN THE SAFETY LOADING TENDS TO ZERO

VSEVOLOD K. MALINOVSKII,* *Steklov Mathematical Institute*

Abstract

When the premium rate is a positive absolute constant throughout the time period of observation and the safety loading of the insurance business is positive, a classical result of collective risk theory claims that probabilities of ultimate ruin $\psi(u)$ and of ruin within finite time $\psi(t, u)$ decrease as $e^{-\varkappa u}$ with a constant $\varkappa > 0$, as the initial risk reserve u increases. This paper establishes uniform approximations to $\psi(t, u)$ with slower rates of decrease when the premium rate depends on u in such a way that the safety loading decreases to zero as $u \to \infty$.

Keywords: Ruin theory; Andersen's risk model; capital-dependent; safety loadings; approximations

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1. Introduction

Insurance is a method of coping with risks, while the object of the theory of risk is to give a mathematical analysis of random fluctuations in an insurance business and to discuss the various means of protection against their inconvenient effects. The classical theory of risk focuses its attention on the *outflow* process, looking first at claim numbers, then at the distribution of claim sizes and finally putting these two together into an aggregate claim amounts process. The *income* process, which is the *initial capital* + *premium income*, is introduced in a rather simple way, growing linearly in time with a constant intensity c. The resulting *surplus process* of the insurance business is generated as *initial capital* + *premium income* − *outflow*.

This bird's eye view of the insurance business has been formalized in the notion of the *collective risk model* introduced by F. Lundberg in 1903. Interpreted by H. Cramér and developed by E. Sparre Andersen and by a number of researchers, it remains one of the main actuarial models concerned with final business results. Ignoring individual policies, this model considers the risk business of an insurance company as a whole: claims occur from time to time and have to be settled by the company, while on the other hand the company receives a continuous flow of risk premiums from the policyholders. An important problem within this model is to investigate the *ruin probability*.

Mathematically, the surplus process at any *operational* time t is described as the *risk reserve process* $R_u(t) = u - \sum_{i=1}^{N(t)} Y_i + ct$ starting at time $t = 0$, where $N(t)$ is the number of claims which have occurred up to time t, $u > 0$ is the initial risk reserve, $c > 0$ is the risk premium rate, $\{T_i\}_{i \geqslant 1}$ are (i.i.d.) interclaim times and $\{Y_i\}_{i \geqslant 1}$ are (i.i.d.) amounts of claims.

The insurer typically needs to charge 'loaded' premiums sufficient for business to take its normal course over a long time. The amount $\tau = c\mathrm{E}(T_1)/\mathrm{E}(Y_1) - 1$, called the *relative safety*

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* Postal address: Steklov Mathematical Institute, Gubkina str 8, 117966 Moscow GSP-1, Russia.
Email address: malinov@mi.ras.ru