Bearing in mind that (see, e.g., Gradshtein and Ryzhik (1980), Eqs. 3.613.3)

$$\int_0^{\pi} \frac{\sin nx \sin x}{1 - 2a\cos x + a^2}\,\mathrm{d}x = \begin{cases} \dfrac{\pi}{2} a^{n-1}, & a^2 < 1, \\ \dfrac{\pi}{2a^{n+1}}, & a^2 > 1, \end{cases}$$

the proof is complete. □

Note that in the case of $\rho < 1$ Eq. (57) follows directly from formulas (5.73) and (5.73a) in Section 5.7 of Riordan (1962); Riordan (1962) attributed this result to Vaulot.

**Lemma 5.2.** *The equalities*

$$\int_0^{\infty} \mathrm{e}^{-bx} I_n(ax)x^{-1}\,\mathrm{d}x = \frac{a^n}{n(b + \sqrt{b^2 - a^2})^n}, \tag{58}$$

$$\int_0^{\infty} \mathrm{e}^{-bx} I_n(ax)\,\mathrm{d}x = \frac{a^n}{\sqrt{b^2 - a^2}(b + \sqrt{b^2 - a^2})^n} \tag{59}$$

*hold true for* $b > a > 0$ *and* $n = 1, 2, \ldots$.

**Proof of Lemma 5.2.** To check (58) and (59), see, e.g., Gradshtein and Ryzhik (1980), Eqs. 6.611.4, 6.623.3 and recall that $I_n(z) = \exp(-n\pi \mathrm{i}/2)J_n(\mathrm{i}z)$, or Watson (1945), Chapter XIII , Section 13.2 , Eqs. (7) and (8) . □

**Lemma 5.3.** *For* $r_n(x) = I_{n+1}(x)/I_n(x)$, *the inequality*

$$0 \leqslant \frac{x}{n + \frac{1}{2} + \left(x^2 + \left(n + \frac{3}{2}\right)^2\right)^{1/2}} \leqslant r_n(x)$$

$$\leqslant \frac{x}{n + \frac{1}{2} + \left(x^2 + \left(n + \frac{1}{2}\right)^2\right)^{1/2}} \leqslant 1$$

*holds true for* $x \geqslant 0$ *and* $n = 0, 1, 2, \ldots$.

**Proof of Lemma 5.3.** This assertion is a particular case (for integer $n$) of the inequality (16) in Amos (1974). □

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