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SURVIVE A DOWNSWING PHASE OF THE UNDERWRITING CYCLE

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AGENDA

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1. Introduction: underwriting cycles due to random surrounding and due to competition

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Long-term variations called "business cycles", are typically common for the most insurers and have several potential causes.

Understanding the driving forces of the underwriting cycles is a paramount theoretical and important practical problem.

▶ Cycles attributed to the fluctuations due to random surroundings, to volatile interest rates, or to random up- and down-swings of the risk exposure in the portfolio. Deficiencies are introduced by the exterior ambiguities limited by the so-called *scenarios of nature*.

• Such fluctuations can not be foreseen and their dynamics is known deficiently since its origin used to be exogenous with respect to the insurance industry.

• It causes inevitable errors in the rate making, and irregularly cyclic underwriting process ensues.

• Adaptive control strategies fighting back cycles due to scenarios of nature were proposed in the multiperiod framework

$$\underbrace{\mathfrak{w}_0 \xrightarrow{\gamma_0} \mathfrak{u}_0 \xrightarrow{\pi_1} \mathfrak{w}_1}_{\text{1-st year}} \cdots \xrightarrow{\pi_{k-1}} \underbrace{\mathfrak{w}_{k-1} \xrightarrow{\gamma_{k-1}} \mathfrak{u}_{k-1} \xrightarrow{\pi_k} \mathfrak{w}_k}_{k\text{-th year}} \cdots .$$

► Cycles <u>attributed to the strategies</u> of aggressive insurers seeking for greater market shares, and by the consequent industry response.

- At the first stage, the response lies in concerted reduction of the rates, sometimes below the real costs of insurance.
- This makes some companies ruined, and agrees with the observation that insurance cycles are correlated with clustered insolvencies.
- For instance (see [Feldblum 2007] with reference on Best's Insolvency Study [Best's 1991]), US industry-wide combined ratios peaked at 109% in 1975 and 117% in 1984. The insurance failure rate, or the ratio of insolvencies to total companies, peaked at 1.0% in 1975 and 1.4% in 1985.
- Insolvencies appear a driving force behind the competition–originated cycles.
- After elimination of the exceedingly aggressive and unwise agents, or just weaker carriers, the prices increase uniformly over the industry.
- The upswing phase of the cycle follows.

2. Price in the years of soft and hard market and portfolio size functions

• The insurance price P^M prevailing in the market is called market price, or market price factor.

• The year of soft market occurs for a particular insurer when the market price factor is below the averaged losses $\mathsf{E}Y$, i.e. as $\mathsf{E}Y > P^M$. The year of hard market for a particular insurer occurs otherwise, i.e. as $\mathsf{E}Y < P^M$.

• The insurer applies maintaining market share control if $P = P^M$. The insurer applies conserving capital control if $P = \mathsf{E}Y$. The insurer applies mixed control, if $P^M < P < \mathsf{E}Y$, as $P^M < \mathsf{E}Y$ (soft market), and $\mathsf{E}Y < P < P^M$, as $\mathsf{E}Y < P^M$ (hard market).

• Without lack of generality[1], the set $\mathcal{P}$ of price controls introduced above may be written as

$$P_\gamma = \gamma P^M + (1-\gamma)\mathsf{E}Y, \quad \gamma \in [0,1],$$

with $P_1 = P^M$ and $P_0 = \mathsf{E}Y$.

[1] In the case of soft market (i.e., $\mathsf{E}Y > P^M$) prices P below P^M cause excessive danger of ruin, while prices P above $\mathsf{E}Y$ yield excessively high rate of elimination of portfolio. Both are claimed unreasonable. The similar arguments are true in the case of hard market.

• For $\gamma \in [0,1]$ and for the insurer's price $P_\gamma \in \mathcal{P}$, the value

$$d_\gamma = P_\gamma - P^M = (1-\gamma)(\mathsf{E}Y - P^M)$$

is called insurer's <u>price deficiency</u> with respect to the market price P^M.

• For $\gamma \in [0,1]$ and for the prices $P_\gamma \in \mathcal{P}$ with deficiency $d_\gamma = P_\gamma - P^M$, introduce the family

$$\mathcal{L} = \{\lambda_{d_\gamma}(s),\ 0 \leqslant s \leqslant t\}$$

of continuous non-negative functions of time, called <u>portfolio size</u> functions.

• Assume that $\lambda_{d_\gamma}(0) = \lambda$. The value λ is referred to as the <u>initial portfolio size</u>.

• In the case of $d_\gamma = 0$ (neutral market or maintaining market share control, $P_1 = P^M$) set $\lambda_{d_\gamma}(s) \equiv \lambda$, $0 \leqslant s \leqslant t$.

• When $d_\gamma > 0$ (soft market and $\gamma \in [0,1)$), the portfolio size functions $\lambda_{d_\gamma}(s)$ must be <u>monotone decreasing</u> in s and $\lambda_{d_{\gamma_1}}(s) < \lambda_{d_{\gamma_2}}(s)$ for all $0 \leqslant s \leqslant t$, as $d_{\gamma_1} > d_{\gamma_2}$.

• When $d_\gamma < 0$ (hard market and $\gamma \in [0,1)$), the portfolio size functions $\lambda_{d_\gamma}(s)$ must be <u>monotone increasing</u> in s and $\lambda_{d_{\gamma_1}}(s) < \lambda_{d_{\gamma_2}}(s)$ for all $0 \leqslant s \leqslant t$, as $d_{\gamma_1} > d_{\gamma_2}$.

3. Portfolio size models in the years of soft and hard market

- Selecting $\mathfrak{L}$, wise is to address to practice.
- [Subramanian 1998], p. 39:

  "Surveys of policyholders have consistently demonstrated some reluctance to switch insurers. In a survey of 2462 policyholders by Cummins et al. [Cummins et al. 1974], 54% of respondents confessed never to have shopped around for auto insurance prices. To the question "Which is the most important factor in your decision to buy insurance?", 40% responded the company, 29% the agent, and only 27% the premium. A similar survey of 2004 Germans (see [Schlesinger et al. 1993]) indicated that, despite the fact that 67% of those responding knew that considerable price differences exist between automobile insurers, only 35% chose their carrier on the basis of their favorable premium. Therefore, we will assume that, given the opportunity to switch for a reduced premium, one-third of the policyholders will do so".

Following that remark, assume that in the year of hard market, i.e. as $d_\gamma > 0$,

$$\lambda_{d_\gamma}(s) = \lambda \cdot r_{d_\gamma}(s), \quad 0 \leqslant s \leqslant t,\ \gamma \in [0,1],$$

where

- $0 \leqslant r_{d_\gamma}(s) \leqslant 1$ is the rate of those who remained in the portfolio by time $s \leqslant t$,
- $m_{d_\gamma}(s) = 1 - r_{d_\gamma}(s)$ is the complementary rate function by time $s \leqslant t$,
- $m_{d_\gamma} = m_{d_\gamma}(+\infty)$ is the ultimate rate of migrants (which does not exceed one-third).

For example, introduce the rate function $r_{d_\gamma}(s)$, $0 \leqslant s \leqslant t$,

- with <u>exponential outgo</u> of migrants,

$$r_{d_\gamma}(s) = \underbrace{(1 - m_{d_\gamma})}_{\text{ultimate remainders}} + \underbrace{m_{d_\gamma}}_{\text{ultimate migrants}} \cdot \underbrace{e^{-s}}_{\text{portion of not yet migrated}} = 1 - \underbrace{m_{d_\gamma}}_{\text{ultimate migrants}} \cdot \underbrace{(1 - e^{-s})}_{\text{portion of just migrated}},$$

which yields

$$\Lambda_{d_\gamma}(t) = \int_0^t \lambda_{d_\gamma}(s)ds = \lambda \cdot t \cdot \underbrace{(1 - m_{d_\gamma})}_{\text{ultimate remainders}} + \lambda \cdot \underbrace{m_{d_\gamma}}_{\text{ultimate migrants}} \cdot \underbrace{(1 - e^{-t})}_{\text{portion of just migrated}}.$$

In most cases the exponential outgo is unrealistically quick. Of more interest may be

- the power rate function

$$r_{d_\gamma}(s) = \underbrace{(1 - m_{d_\gamma})}_{\substack{\text{ultimate}\\ \text{remainders}}} + \underbrace{m_{d_\gamma}}_{\substack{\text{ultimate}\\ \text{migrants}}} \cdot \underbrace{(1+s)^{-k}}_{\substack{\text{portion of not}\\ \text{yet migrated}}} = 1 - \underbrace{m_{d_\gamma}}_{\substack{\text{ultimate}\\ \text{migrants}}} \cdot \underbrace{\big(1 - (1+s)^{-k}\big)}_{\text{power outgo}}, \quad k > 0,$$

which yields

$$\Lambda_{d_\gamma}(t) = \int_0^t \lambda_{d_\gamma}(s) ds = \begin{cases} \lambda t(1 - m_{d_\gamma}) + \lambda m_{d_\gamma}\big(1 - (t+1)^{-k+1}\big)/(k-1), & k \neq 1, \\ \lambda t(1 - m_{d_\gamma}) + \lambda m_{d_\gamma} \ln(1+t), & k = 1. \end{cases}$$

As $k < 1$, the migrating part in the portfolio is slow enough and still influences $\Lambda_{d_\gamma}(t)$ considerably.

- The concept of the set $\mathcal{L}$ of portfolio size functions has to be further developed. For example, it may be sensible to allow dependence of the portfolio size functions on the initial risk reserve[2].

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[2] It is arguable that the outgo of insureds becomes more intensive from e.g., a smaller company, for not to mention such an abstract term as the initial risk reserve. That may be checked by means of a survey of policyholders.

4. Annual risk reserve process and annual probabilities of ruin

Assume that fixed are the families $\mathcal{P}$ of the price controls and $\mathcal{L}$ of the portfolio size functions.

• For $P_\gamma \in \mathcal{P}$ with deficiency d_γ and for the corresponding portfolio size function $\lambda_{d_\gamma} \in \mathcal{L}$, assume that the claim number process is a non-homogeneous Poisson process $\nu_\gamma(s)$, $0 \leqslant s \leqslant t$, with the yield (intensity) function

$$\Lambda_{d_\gamma}(s) = \int_0^s \lambda_{d_\gamma}(z)dz, \quad 0 \leqslant s \leqslant t.$$

• Assume that Y_i, $i = 1, 2, \ldots$, are i.i.d. and independent on the claim number process $\nu_\gamma(s)$, $0 \leqslant s \leqslant t$. The claim outcome process associated with the portfolio size function $\lambda_{d_\gamma} \in \mathcal{L}$ is the compound non-homogeneous Poisson process

$$\sum_{i=1}^{\nu_\gamma(s)} Y_i,$$

as $\nu_\gamma(s) > 0$, or zero, as $\nu_\gamma(s) = 0$, $0 \leqslant s \leqslant t$.

• The <u>premium income</u> process associated with the portfolio size function $\lambda_{d_\gamma} \in \mathfrak{L}$ and with the premium factor P_γ is the non-random process

$$P_\gamma \Lambda_{d_\gamma}(s) = P_\gamma \int_0^s \lambda_{d_\gamma}(z)dz, \quad 0 \leqslant s \leqslant t.$$

• The <u>risk reserve</u> process generated by the premium income process and claim outcome processes is the random process

$$R_{u,\gamma}(s) = u + P_\gamma \Lambda_{d_\gamma}(s) - \sum_{i=1}^{\nu_\gamma(s)} Y_i,$$

as $\nu_\gamma(s) > 0$, or $u + P_\gamma \Lambda_{d_\gamma}(s)$, as $\nu_\gamma(s) = 0$, $0 \leqslant s \leqslant t$. The value $u > 0$ is called the <u>initial risk reserve</u>.

Lemma 1. For a homogeneous Poisson process $N_\lambda(s)$, $0 \leqslant s \leqslant t$, with intensity $\lambda > 0$,

$$R_{u,\gamma}(s) = \hat{R}_{u,\gamma}(\tau(s)), \quad 0 \leqslant s \leqslant t,$$

where $\tau(s) = \Lambda_{d_\gamma}(s)/\lambda$, $0 \leqslant s \leqslant t$, is the <u>operational time</u>, and where

$$\hat{R}_{u,\gamma}(s) = u + [P_\gamma \lambda]s - \sum_{i=1}^{N_\lambda(s)} Y_i, \quad 0 \leqslant s \leqslant \Lambda_{d_\gamma}(t)/\lambda.$$

• The probability

$$\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\}$$

is called annual probability of ruin, or probability of ruin within time t.

THEOREM 1. *In the year of soft market (i.e., as* $\mathsf{E}Y > P^M$*) the probability*

$$\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\}$$

is monotone increasing, as γ *increases.*

• Since $\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) = \inf_{0\leqslant s\leqslant \Lambda_{d_\gamma}(t)/\lambda} \hat{R}_{u,\gamma}(s)$, one has

$$\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\} = \mathsf{P}\Big\{\inf_{0\leqslant s\leqslant \Lambda_{d_\gamma}(t)/\lambda} \hat{R}_{u,\gamma}(s) < 0\Big\}$$

$$= \mathsf{P}\Big\{\inf_{0\leqslant s\leqslant \Lambda_{d_\gamma}(t)/\lambda} \Big(u + \underbrace{[\mathsf{E}Y - \gamma(\overbrace{\mathsf{E}Y - P^M}^{>0})]}_{P_\gamma}\lambda s - \sum_{i=1}^{N_\lambda(s)} Y_i\Big) < 0\Big\}.$$

• In the year of soft market P_γ is monotone decreasing, as γ increases, from $P_0 = \mathsf{E}Y$ to $P_1 = P^M$, with $P_0 > P_1$, and $\Lambda_{d_\gamma}(t)$ is monotone increasing, as γ increases. Both factors contribute to a monotone growth of $\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\}$, as γ increases.

THEOREM 2. *Assume that* Y_i, $i = 1, 2, \ldots$, *are i.i.d. exponential with intensity* μ (*i.e.,* $1/\mu = \mathsf{E}Y$) *and denote by* $I_n(z)$ *the modified Bessel function of* n*th order,* z *real and* $n = 0, 1, 2, \ldots$. *In that model*

$$\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\} = e^{-u\mu} \sum_{n\geqslant 0} \frac{(u\mu)^n}{n\,!} (P_\gamma \mu)^{-(n+1)/2} \times \int_0^{\Lambda_{d_\gamma}(t)} \frac{n+1}{x} e^{-(1+P_\gamma\mu)x} I_{n+1}(2x\sqrt{P_\gamma\mu})\, dx.$$

The alternative expression is

$$\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\} = -\frac{1}{\pi}\int_0^\pi f_t(x,u)\,dx + \begin{cases} (1/P_\gamma\mu)\exp\{-u\mu(1-1/P_\gamma\mu)\}, & P_\gamma\mu > 1,\\ 1, & P_\gamma\mu \leqslant 1,\end{cases}$$

where

$$f_t(x,u) = (P_\gamma\mu)^{-1}(1 + (P_\gamma\mu)^{-1} - 2(P_\gamma\mu)^{-1/2}\cos x)^{-1} \times \exp\left\{u\mu\left((P_\gamma\mu)^{-1/2}\cos x - 1\right) - \Lambda_{d_\gamma}(t)P_\gamma\mu\left(1 + (P_\gamma\mu)^{-1} - 2(P_\gamma\mu)^{-1/2}\cos x\right)\right\} \times \left[\cos\left(u\mu(P_\gamma\mu)^{-1/2}\sin x\right) - \cos\left(u\mu(P_\gamma\mu)^{-1/2}\sin x + 2x\right)\right].$$

5. Admissible risk reserve and premium controls

- In the year of soft market, <u>admissible</u> are those controls which do not compel

(A) the annual probability of ruin be larger than a prescribed value $\alpha \in (0,1)$, and
(B) the year-end portfolio size be less than a prescribed lower limit L.

$$\underbrace{\mathfrak{w}_0 \xrightarrow{\gamma_0} \mathfrak{u}_0 \xrightarrow{\pi_1} \mathfrak{w}_1}_{\text{1st year, } P_1^M, \alpha_1} \cdots \xrightarrow{\pi_{k-1}} \mathfrak{w}_{k-1} \underbrace{\xrightarrow{\gamma_{k-1}} \mathfrak{u}_{k-1} \xrightarrow{\pi_k} \mathfrak{w}_k}_{k\text{th year, } P_k^M, \alpha_k} \cdots .$$

- Admissible risk reserve (annual) controls

- Admissible premium (annual) controls, the solvency point of view (A)

THEOREM 3. *For sufficiently small $\alpha \in (0,1)$, for the initial risk reserve u and for the family $\mathfrak{L}$, in the year of soft market allowed are the price controls $P_\gamma \in \mathcal{P}$, $\gamma \in [0, \gamma_{t,u|\mathfrak{L}}(\alpha)]$, where $\gamma_{t,u|\mathfrak{L}}(\alpha)$ is the <u>unique solution</u> of the equation*

$$\mathsf{P}\{\inf_{0 \leqslant s \leqslant t} R_{u,\gamma}(s) < 0\} = \alpha,$$

as $\mathsf{P}\{\inf_{0 \leqslant s \leqslant t} R_{u,1}(s) < 0\} \geqslant \alpha$, *and* $\gamma_{t,u|\mathfrak{L}}(\alpha) = 1$, *as* $\mathsf{P}\{\inf_{0 \leqslant s \leqslant t} R_{u,1}(s) < 0\} < \alpha$.

• Put $\gamma_{t,\alpha}$ for $\gamma_{t,u|\mathfrak{L}}(\alpha)$, set $\mathsf{P}\{\inf_{0\leqslant s\leqslant t} R_{u,\gamma}(s) < 0\} = \boldsymbol{\psi}_t(\gamma)$ and note that in the year of soft market $\boldsymbol{\psi}_{+\infty}(\gamma) = 1$.

THEOREM 4. *For* $\tau_\gamma = -\gamma(\mathsf{E}Y - P^M)/\mathsf{E}Y$, $\gamma \in (0,1]$, *assume that* $\tau_\gamma < 0$. *Then*[3]

$$\sup_{t\in\mathsf{R}^+} \left|\boldsymbol{\psi}_t(\gamma) - \Phi_{\{0,1\}}((\Lambda_{d_\gamma}(t) - M_{\tau_\gamma} u\mu)/(S_{\tau_\gamma}(u\mu)^{1/2}))\right| = \underline{O}(u^{-1/2}), \quad \textit{as} \;\; u \to \infty,$$

where $M_{\tau_\gamma} = -1/\tau_\gamma$, $S^2_{\tau_\gamma} = -2/\tau^3_\gamma$.

• Introduce $\phi_t(\gamma) = \boldsymbol{\psi}_{+\infty}(\gamma) - \boldsymbol{\psi}_t(\gamma) = 1 - \boldsymbol{\psi}_t(\gamma)$ the probability of <u>ultimate ruin after time</u> t, and rewrite $\phi_t(\gamma_{t,\alpha}) = 1 - \boldsymbol{\psi}_t(\gamma_{t,\alpha}) = 1 - \alpha$, which yields

$$\gamma_{t,\alpha} = \phi_t^{-1}(1-\alpha).$$

THEOREM 5. *For* $\tau_\gamma = -\gamma(\mathsf{E}Y - P^M)/\mathsf{E}Y$, $\gamma \in (0,1]$, *set* $a_\gamma = (1 - \sqrt{1+\tau_\gamma})^2$ *and* $b_\gamma = 1/\sqrt{1+\tau_\gamma}$. *In the framework of Theorem 2, one has* $\tau_\gamma < 0$ *and*

$$\phi_t(\gamma) = \frac{b_\gamma^{3/2}(b_\tau u\mu + 1)}{2\sqrt{\pi} a_\gamma (\Lambda_{d_\gamma}(t))^{3/2}} e^{-u\mu(1-b_\gamma)} e^{-a_\gamma \Lambda_{d_\gamma}(t)} \exp\Big\{ -\frac{b_\gamma^3 (u\mu)^2}{4\Lambda_{d_\gamma}(t)}\Big\}\Big\{1 + \underline{O}(\Lambda_{d_\gamma}^{-1/2}(t))\Big\}$$

for $u \leqslant \underline{O}(\Lambda_{d_\gamma}^{1/2}(t))$, *as* $t \to \infty$.

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[3] Under rather general regularity conditions. The result is suitable to apply for $u \geqslant \underline{O}(\Lambda_{d_\gamma}^{1/2}(t))$, as $t \to \infty$.

• Admissible premium (annual) controls, the portfolio size point of view (B)

THEOREM 6. *For sufficiently small $\alpha \in (0,1)$, for the initial risk reserve u and for the family $\mathcal{L}$, in the year of soft market allowed are the price controls $P_\gamma \in \mathcal{P}$, $\gamma \in [\gamma_L, 1]$, where*

$$\gamma_L = \inf\{\gamma \in [0,1] : \lambda_{d_\gamma}(t) = L\} > 0,$$

as $\lambda_{d_0}(t) < L$, and $\gamma_L = 0$, as $\lambda_{d_0}(t) \geqslant L$.

• Theorems 3–6 yield the set of the annual price controls allowed both from (A) solvency and (B) portfolio size points of view. This set is

$$P_\gamma \in \mathcal{P}, \quad \gamma \in [0, \gamma_{t,u|\mathcal{L}}(\alpha)] \cap [\gamma_L, 1] = [\gamma_L, \gamma_{t,u|\mathcal{L}}(\alpha)].$$

6. Conclusion: a strategy beating the downswing phase of the cycle

For the family $\mathfrak{L}$ and for a sequence $u, w_1, \ldots, w_{k-1}$ of the initial risk reserve values, as the $(i-1)$st year-end risk reserve is assumed equal to the initial risk reserve in ith year ($i = 2, \ldots, k$), the adaptive control strategy beating the downswing phase of the insurance cycle with the period k, generated by the market prices $P_1^M > \cdots > P_k^M > 0$, all below the average risk $\mathsf{E}Y$, is

$$\begin{array}{lll} P_1(u) = P_\gamma, \quad \gamma \in [\gamma_L, \gamma_{t,u|\mathfrak{L}}(\alpha_1)], & \text{if} & [\gamma_L, \gamma_{t,u|\mathfrak{L}}(\alpha_1)] \neq \varnothing, \\ P_2(w_1) = P_\gamma, \quad \gamma \in [\gamma_L, \gamma_{t,w_1|\mathfrak{L}}(\alpha_2)], & \text{if} & [\gamma_L, \gamma_{t,w_1|\mathfrak{L}}(\alpha_2)] \neq \varnothing, \\ \cdots\cdots & & \\ P_k(w_{k-1}) = P_\gamma, \quad \gamma \in [\gamma_L, \gamma_{t,w_{k-1}|\mathfrak{L}}(\alpha_k)], & \text{if} & [\gamma_L, \gamma_{t,w_{k-1}|\mathfrak{L}}(\alpha_k)] \neq \varnothing. \end{array}$$

Recall that $\alpha_1, \ldots, \alpha_k$ in

$$\mathfrak{w}_0 \underbrace{\xrightarrow{\gamma_0} \mathfrak{u}_0 \xrightarrow{\pi_1} \mathfrak{w}_1}_{\text{1st year, } P_1^M, \alpha_1} \cdots \xrightarrow{\pi_{k-1}} \mathfrak{w}_{k-1} \underbrace{\xrightarrow{\gamma_{k-1}} \mathfrak{u}_{k-1} \xrightarrow{\pi_k} \mathfrak{w}_k}_{k\text{th year, } P_k^M, \alpha_k} \cdots,$$

are the allowed levels or ruin within the downswing phase of the underwriting cycle.